A Passenger Knock-On Delay Model for Timetable Optimisation

Peter Sels[123], Thijs Dewilde[1], Dirk Cattrysse[1], Pieter Vansteenwegen[1]
[1]KU Leuven, University of Leuven
[2]Logically Yours BVBA, Antwerp
[3]Infrabel, Department of Rail Access, Brussels

Abstract

In the process of timetable creation, sufficient time should be scheduled between any pair of trains using a common infrastructure section in order to avoid that a delay on the first train will cause a delay on the second train too. However, when this time buffer becomes very high, the positive incremental buffering effect diminishes and other negative effects may appear, like reduced timetable efficiency or lower than optimal remaining time between the other trains on the same infrastructure resource. This means there is a trade-off to make. We make this trade-off by analytically deriving the knock-on delays as passengers experience it in practice and by including these delays in our goal function: total expected passenger journey time in practice.

We use this goal function in our Mixed Integer Linear Programming (MILP) model to optimise from scratch, the timetable of all 203 hourly passenger trains in Belgium. We then also compare our resulting timetable with the original schedule, and conclude that both the knock-on component as well as the total expected passenger time are reduced.

Keywords: Knock-On Delay Model, Expected Passenger Time, Integer Linear Programming, Goal Function

1 Introduction

A railway timetable can be aptly represented by a graph. Graph vertices are train arrival and departure times. The graph's edges are either primary edges representing intra-train actions: ride and dwell, or secondary edges, representing inter-train actions: transfer or turn-around. Other secondary edges represent a required time difference: headway requirements [Kro09]. For all edges, primary or secondary, a minimum time is required and we also add a non-negative supplement. Note that we use the term *supplement* also in the meaning of *buffer* between two trains on a common infrastructure resource. The purpose of the supplements can be twofold. Firstly, they are sometimes needed as slack between two already planned timetable times. Indeed, imagining that one would plan the primary edges first, some slack would result for the inter-train transfer, turn-around and headway edges. Secondly, a larger than slack-only supplement could be needed to make a timetable robust against delay. However, supplements may also not become too large, resulting in trains riding too slow or idling too much and as such resulting in an inefficient planning. So, obviously there is a trade-off, per supplement, between robustness and efficiency. Additionally, when edges are part of a common graph cycle, the sum of minimum process times and supplements over all edges of the cycle have to sum up to a multiple of the timetable period [Gov10]. This means choices of supplements of these edges are related and one also has to be able to properly weigh the costs and benefits of the supplement choices on different edges. We consider one train more important than another when it has more passengers present on it. We could introduce artificial train class priorities, but prefer to directly weigh importance with passenger numbers instead. In [Sel11] we derived passenger numbers on all trains at all locations, starting from ticket sales data. With this information, we can formulate the *total expected passenger time in practice* [Dew11; Sel13a] as a function of the timetable. More specifically, it is a function of 3 parameters sets: (1) the action minima, (2) the assumed primary delays and (3) the planned supplements. Secondary delays also increase this expected passenger time, but are itself a function of the three mentioned parameter sets. The resulting total function is to be minimised to generate an optimal timetable for passengers. The minima are fixed, so in each timetable it will generate the same amount of expected time. The supplements are the decision variables of the timetable, so given the delay assumptions, their values determine any quality criterium of the timetable as expected passenger time, robustness and efficiency.

The total expected passenger time has been analytically derived as a function of minima and supplements in [Sel13a] for departing, through, transfer and arriving passengers. In this paper we add the derivation of the knock-on delay as a function of the minimum and supplement present on a headway edge. Indeed both a headway minimum time as well as a knock-on delay should be modelled whenever two trains on a common resource occur. So a *hard* headway constraint and a *soft* knock-on cost as a term in the goal function are always modelled on the same edge.

Section 2 lays out an analytical derivation of the knock-on delay function. Section 3 presents the results obtained when using these knock-on delay functions as terms in the goal function for a system of all 203 trains currently departing between 7 and 8 am in the cyclic Belgian timetable. Section 4 draws conclusions and hints at some further work.

2 Knock-On Delay Derivation

When train i is riding or dwelling on a track and it gets delayed, it can delay train j which follows it on the same track. We will derive a cost function that gives us the expected delay for all passengers on the second train as a function of the planned time in between the two trains and the expected delays on these trains.

We define the number of passengers on train i as f_i and on train j as f_j. As [Sel13b] explains, for trains riding in the same direction on a common track, headway edges exist between both the vertices representing the beginning of the trains' ride actions in both directions, cyclically and also between the endings of the trains' ride actions again in both directions, cyclically. For trains riding in opposite directions on a common track, a headway edge exists between the end of the first train's ride action and the beginning of the other train's ride action and vice versa, cyclically. In the sequel, when we mention a knock-on edge between train i and j, we more specifically mean the knock-on edge between two vertices v_i and v_j, where these vertices can be a begin or end vertex of a ride edge.

We can suppose the vertices v_i and v_j, which represent event times, to experience primary delays according to (commonly used [Han08]) negative exponential distributions

$$p_i(x) = a_i e^{-a_i x}, p_j(y) = a_j e^{-a_j y}, \tag{1}$$

where x and y are the primary delays of time points v_i and v_j and $p_i(x)$ and $p_j(y)$ their respective probabilities. The *expected* delays of these distributions are calculated to be

$$\overline{c_i} = \int_0^\infty x a_i e^{-a_i x} dx = \frac{1}{a_i}, \overline{c_j} = \int_0^\infty y a_j e^{-a_j y} dy = \frac{1}{a_j}. \tag{2}$$

Say that, on top of the mandatory heading time h between trains i and j, which has to be respected at any time, there is a *planned* supplement time $s_{i,j}$ and similarly a *planned* supplement $s_{j,i}$ between trains j and i. Then, the probability that due to combined delays of trains i and j, one train will delay the other is calculated by adding all cases where the delay difference of both trains exceeds the supplement between them, weighting these cases with the probability that they occur. This is done by integrating over a triangle area where the delay difference $x - y \geq s_{i,j}$ so $x \geq y + s_{i,j}$ and over another where $y \geq x + s_{j,i}$ as in

$$\begin{aligned} p_{x \geq y+s_{i,j}}(a_i, a_j, s_{i,j}) &= \int_0^\infty \int_{y+s_{i,j}}^\infty a_i e^{-a_i x} \cdot a_j e^{-a_j y} dx dy = \tfrac{a_j e^{-a_i s_{i,j}}}{a_i + a_j}, \\ p_{y \geq x+s_{i,j}}(a_i, a_j, s_{j,i}) &= \int_0^\infty \int_{x+s_{j,i}}^\infty a_i e^{-a_i x} \cdot a_j e^{-a_j y} dy dx = \tfrac{a_i e^{-a_j s_{j,i}}}{a_i + a_j}. \end{aligned} \tag{3}$$

In the area where $x < y + s_{i,j}$ and $y < x + s_{j,i}$, $s_{i,j}$ respectively $s_{j,i}$ are large enough to absorb primary delays and avoid knock-on delays. The total *expected* knock-on delay of train i on train j is calculated by multiplying, for each case where a knock-on delay occurs, its probability, with the knock-on delay amount occurring and then integrating these products over the same triangular integration areas as before. Via partial integration, one can prove

$$\begin{aligned} tKO_{i,j}(a_i, a_j, s_{i,j}) &= \int_0^\infty \int_{y+s_{i,j}}^\infty a_i e^{-a_i x} \cdot a_j e^{-a_j y}(x - y - s_{i,j}) dx dy = \tfrac{a_j e^{-a_i s_{i,j}}}{a_i(a_i + a_j)}, \\ tKO_{j,i}(a_i, a_j, s_{j,i}) &= \int_0^\infty \int_{x+s_{j,i}}^\infty a_i e^{-a_i x} \cdot a_j e^{-a_j y}(y - x - s_{j,i}) dy dx = \tfrac{a_i e^{-a_j s_{j,i}}}{a_j(a_i + a_j)}. \end{aligned} \tag{4}$$

From equations (4), two properties can be derived. First, the larger the planned separation time $s_{i,j}$ between the trains, the lower $tKO_{i,j}$, so the lower the expected knock-on delay on train j. Second, the lower the expected primary delay $\overline{c_i} = 1/a_i$ on train i, the higher a_i, the lower $tKO_{i,j}$, so the lower the expected knock-on delay on train j. These tendencies are indeed what we expect in practice as well. Since we are interested in the knock-on delays as *passengers* experience them in practice, we multiply the train knock-on delay with the number of passengers on the knocked-on train and get

$$\begin{aligned} pKO_{i,j}(a_i, a_j, s_{i,j}) &= f_j \cdot tKO_{i,j} = f_j \cdot \tfrac{a_j e^{-a_i s_{i,j}}}{a_i(a_i+a_j)}, \\ pKO_{j,i}(a_i, a_j, s_{j,i}) &= f_i \cdot tKO_{j,i} = f_i \cdot \tfrac{a_i e^{-a_j s_{j,i}}}{a_j(a_i+a_j)}. \end{aligned} \tag{5}$$

If only two trains i and j are to be planned on a common resource, in a one hour period, what are the ideal supplement times $s_{i,j}, s_{j,i}$ to be planned in between them? This will depend on their passenger numbers f_i, f_j and their expected delays a_i and a_j. First, note that there is a relation to respect between $s_{i,j}$ and $s_{j,i}$. Indeed, the constraint for the cycle formed by the two headway edges between trains i and j is

$$h + s_{i,j} + h + s_{j,i} = T \text{ or equivalently } s_{j,i} = T - 2h - s_{i,j}. \tag{6}$$

After substitution of $T - 2h - s_{i,j}$ for $s_{j,i}$ in $pKO_{j,i}$, $pKO_{j,i}$ is clearly a function of $s_{i,j}$. Since $pKO_{i,j}$ and $pKO_{j,i}$ are both convex functions of $s_{i,j}$, their sum is a convex function of $s_{i,j}$ as well. This means the optimal spreading of two trains per time period T can be calculated by minimising the total expected delay on all passengers of both trains as

$$\begin{aligned} 0 &= \tfrac{d}{ds_{i,j}} \left(pKO_{i,j} + pKO_{j,i}\right) \\ \Leftrightarrow 0 &= \tfrac{d}{ds_{i,j}} \left(f_j \cdot \tfrac{a_j e^{-a_i s_{i,j}}}{a_i(a_i+a_j)} + f_i \cdot \tfrac{a_i e^{-a_j(T-2h-s_{i,j})}}{a_j(a_i+a_j)}\right) \\ \Leftrightarrow 0 &= -f_j \cdot \tfrac{a_j e^{-a_i s_{i,j}}}{a_i+a_j} + f_i \cdot \tfrac{a_i e^{-a_j(T-2h-s_{i,j})}}{a_i+a_j} \\ \Leftrightarrow f_j \cdot a_j e^{-a_i s_{i,j}} &= f_i \cdot a_i e^{-a_j(T-2h-s_{i,j})} \\ \Leftrightarrow ln\left(\tfrac{f_j \cdot a_j}{f_i \cdot a_i}\right) &= -a_j(T - 2h - s_{i,j}) + a_i(s_{i,j}) \\ \Leftrightarrow s_{i,j} &= \tfrac{a_j(T-2h)+ln\left(\frac{f_j a_j}{f_i a_i}\right)}{a_i+a_j} \end{aligned} \tag{7}$$

It follows from symmetry that

$$s_{j,i} = \frac{a_i(T - 2h) + ln\left(\frac{f_i a_i}{f_j a_j}\right)}{a_i + a_j}. \tag{8}$$

The right hand sides of equations (7) and (8) sum up to $T - 2h$ as equation (6) requires.

As an example, for $T = 60$ minutes and $h = 3$ minutes, a train i with an expected delay of $1/a_i = 3$ minutes and $f_i = 100$ passengers on it and a train j with an expected delay of $1/a_j = 1$ minute and $f_j = 300$ passengers, would be spread according to equations (7) and (8) as $s_{i,j} = \frac{1(60-2\cdot3)+ln(300\cdot1/(100\cdot1/3))}{1/3+1} = 42.15$ minutes and $s_{j,i} = \frac{1/3(60-2\cdot3)+ln(100\cdot1/3/(300\cdot1))}{1/3+1} = 11.85$ minutes and indeed as equation (6) requires $42.15 + 3 + 11.85 + 3 = 60$ minutes.

This kind of balancing of supplements between trains on the same resource will be done by our solver when we add the costs in equation (5) to the goal function. (Note that also choices of supplements on graph edges in common cycles can affect the choice of $s_{i,j}$ and

$s_{j,i}$ and vice versa.) We take the approach of generating all knock-on costs between *all* train pairs using the same infrastructure resource, irrespective of their order. This has two reasons. First, unlike the method where we add only knock-on costs between directly subsequent trains, this method works without relying on the yet unknown order of trains. Second, suppose trains i, j and k follow each other in this order on a resource and train i has a large expected primary delay $1/a_i$, train j has a small $1/a_j$ but has very few people f_j on it while train k has a lot of people f_k on it. Then $pKO_{i,j}$ and $pKO_{j,k}$ can be small for low $s_{i,j}$ and low $s_{j,k}$, allowing the three trains, ordered as i, j, k, to be scheduled close together in time, even though $pKO_{i,k}$ will then be large. The fact that cases where $pKO_{i,k} \gg pKO_{i,j} + pKO_{j,k}$ can occur, shows that $pKO_{i,k}$ has to be added to capture all potential knock-on costs.

For N trains using the same resource during every timetable period T cyclically, this method generates $N \cdot (N-1)$ knock-on terms in the goal function. For each resource R, we define the index set I_R as the set of indices of trains that use R. Then, according to equation (5), the total knock-on cost pKO_R for all trains which use resource R is

$$\forall R : pKO_R = \sum_{\substack{i,j \in I_R \\ i \neq j}} f_j \cdot \frac{a_j e^{-a_i s_{i,j}}}{a_i(a_i + a_j)}. \tag{9}$$

For evaluation of the knock-on cost of a given schedule or for non-linear optimisation, equation (9) can be directly used. For a linear solver though, we need to linearise it first. Since each of the terms in (9) is convex in the variable $s_{i,j}$, we can use a standard linearisation trick for convex cost functions. This entails two steps. First, for each of the terms, we define a helper variable $pKO_{R,i,j}$ and impose on them

$$\forall R : \forall_{\substack{i,j \in I_R \\ i \neq j}} : pKO_{R,i,j} \geq f_j \cdot \frac{a_j e^{-a_i s_{i,j}}}{a_i(a_i + a_j)}. \tag{10}$$

All helper variables $KO_{R,i,j}$ are added to the global goal function of expected passenger time. Units match. Since we *minimise* our global goal function, all $KO_{R,i,j}$ are pushed down so that they will be equal to instead of greater than the right hand side of equation (10). Second, the right hand side of (10) is replaced by a number of line segments approximating it. Here, we use 2 segments. So for each $KO_{R,i,j}$ term, we define three points

$$\forall R : \forall_{\substack{i,j \in I_R \\ i \neq j}} : \begin{cases} (s_{i,j,0}, ko_{i,j,0}) = (0, f_j \cdot \frac{a_j}{a_i(a_i+a_j)}) \\ (s_{i,j,1}, ko_{i,j,1}) = (T/15, f_j \cdot \frac{a_j e^{-a_i T/15}}{a_i(a_i+a_j)}) \\ (s_{i,j,2}, ko_{i,j,2}) = (T, f_j \cdot \frac{a_j e^{-a_i T}}{a_i(a_i+a_j)}). \end{cases} \tag{11}$$

The low and high end of the segments are 0 and T so that the whole supplement range is covered. We use $T/15$, or 4 minutes for T equal to one hour, as the abcis of the middle point, because, in our tests, this resulted in the closest approximation to the curve $KO_{R,i,j}$ for most practical cases. Then, with these known values, equation (10) is linearised to

$$\forall R : \forall_{\substack{i,j \in I_R \\ i \neq j}} : \begin{cases} pKO_{R,i,j} & \geq & ko_{i,j,0} + \frac{ko_{i,j,1} - ko_{i,j,0}}{s_{i,j,1} - s_{i,j,0}} \cdot (s_{i,j} - s_{i,j,0}) \\ pKO_{R,i,j} & \geq & ko_{i,j,1} + \frac{ko_{i,j,2} - ko_{i,j,1}}{s_{i,j,2} - s_{i,j,1}} \cdot (s_{i,j} - s_{i,j,1}) \end{cases} \tag{12}$$

We add all $pKO_{R,i,j}$ as variables to our goal function and add the inequalities (12) with the values calculated as in (11) as hard constraints to our MILP model. As such, we have extended our model with a method that accounts for knock-on delays in a way that is properly balanced with the other goal function terms. Note that the obtained estimation of passenger knock-on delay cost can also be used in other than timetable optimisation models. A linear optimisation model maximising capacity consumption with the goal of capacity estimation, as for example [Mus13], could forbid or penalise scenarios with too much knock-on delay.

3 Optimisation Results

For all 203 hourly passenger trains in Belgium, departing between 7 and 8 am in the timetable of June 12th 2013, visiting 1770 open line track sections and calling at all 550 stations, the macroscopic model of constraints as described in [Sel13b] has been set up. (Overtaking is only allowed in stations with 4 or more platform tracks.) The goal function - expected passenger time in practice - as described in [Sel13a] and now extended with the cost terms for knock-on delays, as derived here in section 2, has been constructed. For each ride and dwell action we assumed varying primary delay distributions with an average of $a\%$ of each action's minimum time. a is given in column 1 of table 1. We compare properties of the original and optimised timetable in the next sections.

3.1 Feasibility: A Solution is Always Returned

Since our model has a goal function that properly penalises the choice of big supplements in a soft yet passenger optimal way, there is no reason for us to add a hard constraint that restricts supplements to any arbitrary value lower than $T - \delta$, where δ is the time resolution of the timetabling model. Other research groups (e.g. Delft [Spa13], e.g. Rotterdam [Kro09]) lack a goal function that automatically restricts *all* supplements and so have to enforce lower more arbitrary upper bounds as a hard constraint on their supplements. As a result they sometimes struggle with infeasibility of their model. We believe we have resolved this issue.

3.2 Quality: The Solution has Lower Expected Passenger Time in Practice

We assume for each action, a primary delay distribution with an average of 2% of the action minimum time. This value of a is Infrabels current best estimate for morning peak hours. Similarly, [Gov07] also uses percentages between 0 to 5%.

Consider figure 1 and its caption. The left half of the figure shows the planned train time total minima and total supplements, both for the *oRiginal* timetable (R) and for the *oPtimised* timetable (P). The right half represents passenger weighted planned time for all

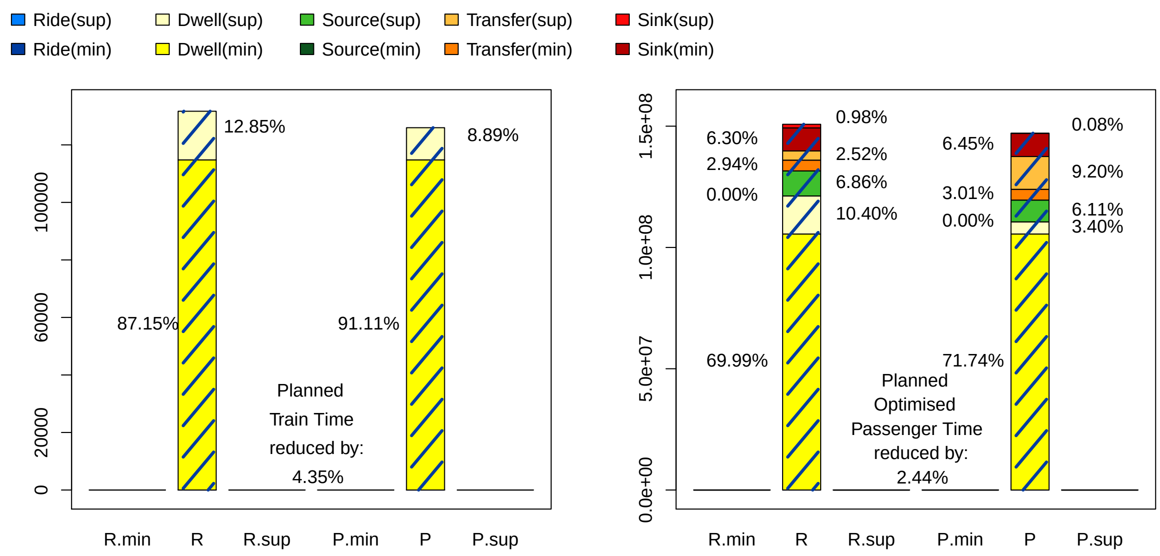

Figure 1: The *planned* time domain. The left half shows total planned train time for all trains. The right half show total passenger time for all passengers. In each box, the left bargraph shows a quantity for the *oRiginal* timetable while the right half shows the same quantity for the *oPtimised* timetable. *min* = sum of all minima, *sup* is sum of all supplements. The sum is not weighted for train time and passenger weighted for passenger time. *Source* corresponds to boarding passengers and *sink* to alighting passengers. In this *planned* domain, the shading with blue lines indicates that these actions were *summed* with ride actions.

origin-destination passenger streams with at least 50 people, again both for original and optimised timetable. There are dark and light versions of some colours (e.g.: yellow, orange). The dark colour indicates the sum of minimum times, while the lighter version indicates the sum of supplement times. The left half of figure 1 shows a decrease of total planned train supplements from 12.85% down to 8.89%. This train time supplement reduction is advantageous for the operator, since, if total train trip time now becomes less than the next lower multiple of hours, the same hourly service can be operated with one less train. [Lie07] also gave an example of this, optimising the Berlin Underground timetable.

The right hand side of figure 1 shows that the planned *passenger* weighted time reduction is a much more pronounced one, from 10.40% down to 3.40% of the same ride+dwell supplements. This is the case because they are now weighted by number of passengers.

In figure 2, instead of planned time, we show expected time, which includes primary delays and their consequences like secondary delays and missed transfers. The left half again represents train time. The right half shows passenger weighted time. The top row is the linear approximation of time as used in the optimisation model. The bottom row shows the actual non-linear time as it is evaluated post-optimisation. The same advantageous stronger supplement reduction in column 2 compared to column 1 is also present in this figure. This is the case for ride+dwell supplements but also for knock-on time. The knock-on compo-

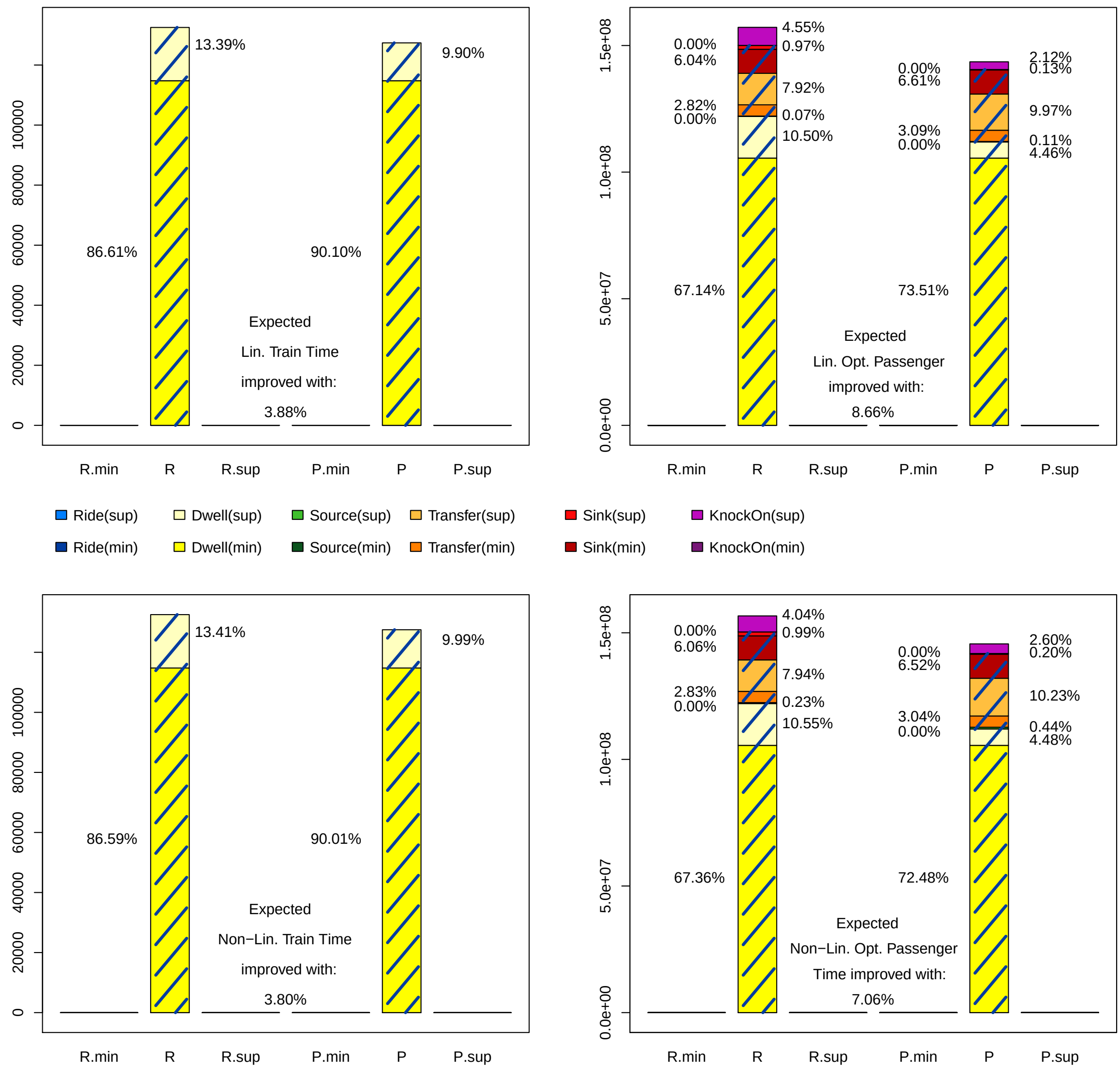

Figure 2: The *expected* time domain. Left and right are train and passenger time as in figure 1. The bottom row shows the non-linear time as used during evaluation. The top row represents the linearised approximation of it as is used during optimisation. So row 1, column 2 shows the totals achieved by optimisation of the goal function. In this *planned* domain, blue line shading indicates these actions were *convoluted* with ride actions. All figures show the case $a = 2\%$ as also reported in table 1.

nent, shown as the top (purple) rectangle of the bar graphs, is reduced in percentage of the total expected passenger time from 4.55% in the original schedule to 2.12% in the optimised schedule. This is for the linearised function as used in optimisation (column 2, row 1). For the non-linearised function (column 2, row 2), post-optimisation evaluation results in a reduction from 4.04% to 2.60%. In both cases, in absolute terms, we more than halve the amount of total expected passenger knock-on delay. The solver achieves this goal by changing orders of trains on common resources and optimally choosing headway supplements, weighing with passenger numbers and also balancing these with other goal function terms. Note that our model assumes the absence of dispatching interventions but with fewer knock-ons happening, the number of necessary dispatching interventions will be lower than in the original timetable as well.

The decrease of the ride+dwell and knock-on times is compensated only slightly by the small increase in expected transfer time. In column 2, representing evaluation on all origin-destination flows of 50 and more passengers, the total time net reduction is 8.66% (row 1, linear) and 7.06% (row 2, non-linear). The fact that the two pictures in column 2 are quite similar, demonstrates that our linearisation, even if only using 2 segments, is effective.

When we evaluate on *all* passenger streams, also the ones with fewer than 50 passengers, the result is a less grand, but still positive 0.42% reduction (non-linear). Plotting distributions of planned passenger journey time versus number of people, we saw that distributions corresponding to the major flows of column 2 are more realistic than the ones corresponding to all passenger streams. None of the major passenger flows, but a minority of the smaller ones have journey times between 2 and 3 hours for a single trip. Some of these are caused by an overenthusiastic diffusion of the zone-OD matrix to the station level [Sel11]. These travellers would most likely prefer other modes of transport. So we consider 7.06% to be our best prediction for reduction of total expected passenger time. Note that an average planned buffer of 8.89% is not enough to totally eliminate all knock-on delays, even though the assumed primary delays have only an average of 2%, seen in train time. The non-zero spread in the primary distribution explains this.

Table 1: Increasing primary delays, characterised by their average of $a\%$ of minimum dwell and ride times. The first column shows $a\%$. Column 2 and 3 show the computation time and the MILP gap achieved. We ran Gurobi 5.5.0 on an Apple MacBook Pro with 2.6GHz Intel i7 processor and 16GB 1.6GHz DDR3 memory. For the first set of result rows, the gap desired was set slightly above what was obtained as the gap of the first returned solution in earlier trials. The results in the last row are obtained by reduction of the desired gap by 1% compared to the first row. Graph size: 203 hourly trains, 5355 ride, 5152 dwell, 17553 major transfer, 31696 knock-on and 166 turn-around edges. Model size: 42609 supplement decision variables, 49415 integer decision variables, 41128 goal function terms for major flows and 58441 evaluation function terms for all flows.

a	solver time	MILP gap	major flows linearised ko-time reduction	major flows linearised time reduction	major flows non-linearised time reduction	all flows linearised time reduction	all flows non-linearised time reduction	missed transfer probability orig.	opt.
%	min.	%	%	%	%	%	%	%	%
2	95	76.2	57	8.66	7.06	1.71	0.42	14.1	2.2
4	43	71.0	52	6.61	4.42	0.84	-1.41	14.6	4.2
6	75	61.3	63	7.65	5.73	2.07	0.13	15.1	1.8
8	66	61.3	59	5.83	3.86	0.40	-1.61	15.6	4.4
2	112	72.6	66	10.58	9.19	2.54	1.31	14.1	2.6

As table 1 shows, compared to the current timetable, our optimised timetables have quite some advantages. First, they respect all minimum ride- and dwell-times and all headway time buffers of 3 minutes between all train pairs on the same track section. In the original timetable sometimes minimum run times and headway times are not respected. Second, we calculated that, over all primary delay assumptions of table 1, the average chance of missing a transfer in the current timetable is at least 14.1% while in our optimised timetables it is at most 4.4%. Depending on the primary delays assumed, in our timetables the expected passenger times are between 7.06% and 0.42% lower than in the original schedule. This decrease is significant, because, of the total passenger time, the irreducible part of minimal ride and dwell times already consumes 67% in the original and 73% in the optimised timetable.

3.3 Computation Speed: The Solution is Returned Quickly

Using the solver abstraction part of the software library milp-logic [Sel12], which we developed and open sourced, as shown in table 1, Gurobi 5.5.0 was able to return a solution for the whole train set, for any primary delay distributions assumed, within about one hour. This is a big improvement compared to the current manual timetabling process that takes many human planners many months.

4 Conclusions and Further Work

This paper has three main contributions. Firstly, we analytically derived the expected passenger time experienced due to knock-on delays as a function of (i) the headway minima, (ii) the chosen headway supplements in a timetable and (iii) expected train delays and linearised this function, so that it can be used for linear optimisation. Secondly, we used the linearised functions as a method to minimise secondary delays, together with other expected passenger time, in a system containing all hourly trains in Belgium. Our results show that we can more than halve the amount of expected passenger knock-on delay in practice. Also, even with addition of many terms to the goal function, optimisation times for the Belgian timetable are only about one hour. Supposing primary delay distributions with an average of 2% of the minimal time of their corresponding actions, our improved timetable reduced expected passenger time for realistic passenger streams by 7.06% compared to the current one. Finally, although restricting the search space and using curtailed goal functions are the easy way to try to reduce solver time, we show that defining an all-encompassing goal function and searching the full solution space can lead to more desirable results: guaranteed feasibility, optimality and even lower solver times.

As for further work, we want to reduce our MILP gap, refine our minimum transfer time differentiating it by station and calibrate our primary delay distributions with train and location specific delays measured in practice. Also, instead of the *frequency-arc* hard constraint approach [Spa13], we want to add terms to the goal function that are due to *uneven spreading over the timetable period* of alternative trains between the same source and destination.

References

[Dew11] T. Dewilde, P. Sels, D. Cattrysse, and P. Vansteenwegen. "Defining Robustness of a Railway Timetable". In: *Proceedings of 4th Int'l Seminar on Railway Operations Modelling and Analysis (IAROR):February 16-18, Rome, Italy.* (2011).

[Gov07] R. M. Goverde. "Railway timetable stability analysis using max plus algebra". In: *Transportation Research Part B: Methodological* 41 (2007), pp. 179–201.

[Gov10] R. M. Goverde. "A delay propagation algorithm for large-scale railway traffic networks". In: *Transportation Research Part C* 18 (2010), pp. 269–287.

[Han08] I. A. Hansen and J. Pachl. *Railway Timetable and Traffic: Analysis, Modelling, Simulation*. Vol. 1. Eurailpress, Hamburg, Germany, Jan. 2008, pp. 1–288.

[Kro09] L. Kroon, D. Huisman, E. Abbink, P.-J. Fioole, M. Fischetti, G. Maróti, A. Schrijver, and R. Ybema. "The New Dutch Timetable: The OR Revolution". In: *Interfaces* 39 (2009), pp. 6–17.

[Lie07] C. Liebchen. "Periodic Timetable Optimization in Public Transport". In: *Operations Research Proceedings* 2006 (2007), pp. 29–36.

[Mus13] L. Mussone and R. Wolfler Calvo. "An analytical approach to calculate the capacity of a railway system". In: *EJOR* 228 (2013), pp. 11–23.

[Sel11] P. Sels, T. Dewilde, D. Cattrysse, and P. Vansteenwegen. "Deriving all Passenger Flows in a Railway Network from Ticket Sales Data". In: *Proceedings of 4th Int'l Seminar on Railway Operations Modelling and Analysis (IAROR): RailRome2011, February 16-18, Rome, Italy.* (Feb. 2011).

[Sel12] P. Sels. *milp-logic: a C++ MILP Solver Abstraction Layer with a C++ Boolean Modelling Layer on Top.* Sept. 2012.

[Sel13a] P. Sels, T. Dewilde, D. Cattrysse, and P. Vansteenwegen. "Expected Passenger Travel Time as Objective Function for Train Schedule Optimization". In: *Proceedings of 5th International Seminar on Railway Operations Modelling and Analysis (IAROR): RailCopenhagen2013, May 13-15, Copenhagen, Denmark.* (May 2013).

[Sel13b] P. Sels, T. Dewilde, D. Cattrysse, and P. Vansteenwegen. "Reducing the Passenger Travel Time in Practice by the Automated Construction of a Robust Railway Timetable". In: *to be submitted soon* (2013).

[Spa13] D. Sparing, R. M. Goverde, and I. A. Hansen. "An Optimization Model for Simultaneous Periodic Timetable Generation and Stability Analysis". In: *Proceedings of 5th International Seminar on Railway Operations Modelling and Analysis (IAROR): RailCopenhagen2013, May 13-15, Copenhagen, Denmark.* (2013).

Corresponding author: Peter Sels, KU Leuven, University of Leuven, Centre for Industrial Management/Traffic & Infrastructure, 3001 Leuven, Belgium, phone: +32 486 95 67 97, e-mail: sels.peter@gmail.com