Expected Passenger Travel Time as Objective Function for Train Schedule Optimization

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1 Necessity of an Objective Function

Most literature in train schedule optimization concentrates on the constraints of the problem. A few examples are [4, 5, 6, 7, 8]. Very little is found on the optimisation criterium: the objective function [2, 3]. We believe one of the main reasons is that in generating schedules that are robust against delays, this is the harder part of the mathematical modelling. Indeed, unlike the hard constraints, it requires the use of statistics, namely delay statistics. It also requires accurate measurement of these delays or derivation from train logs. We consider total passenger travel time as the most important criterium for judging the quality of a passenger train schedule [1, 9, 10, 11], so this is what we want to minimize as the objective function. Approaches to train scheduling that lack an objective function like this are unable to make an informed decision about the size of supplements. These methods usually resort to inserting buffers on a train traject of up to some maximum percentage of the total train travel time [5]. Such approaches do not take into account the typical delays at a certain location, nor the number of passengers sitting in, entering, leaving or transferring to or from this train at that location. Therefore, it is impossible to calculate the travel time increase or decrease resulting from a buffer on all passenger streams.

In our objective function, we use passenger numbers instead of trains as weight factors. While optimizing a schedule, we focus on decreasing this objective function and this implies that no or few planned transfers are missed, especially if many people are transferring. Not only transfers, but also ride and dwell actions should be planned with supplements that are neither too small such that they are unable to absorb any practical delay, nor too big such that they create unnecessary idle time in practice. Thanks to our detailed objective function, we can make this trade-off and determine optimal buffer times which result in better schedules, even optimal ones for passengers.

2 Derivation of the Objective Function

In our paper, we analytically derive the stochastically expected passenger time on each location throughout the network.

Firstly, we decomposed a general train network action graph into actions of five distinguished types: ride and dwell train actions and the source (embarking), transfer and sink (alighting) passenger actions and then identified four types of subsequent action-pairs, representing separate passenger streams: departing, through, transfer and arriving passengers. Then, we also take knock-on delays into account, where only the passenger numbers on the second train matter.

Thanks to this, we were able to analytically derive the probability and cost functions for each of these local passenger flows. This includes the probabilistic trade-off for catching or missing a next action. These local functions of one or two supplement variables, indicate the cost trade-off and can be used to evaluate a given schedule on quality of these local supplements and even determine the values of the optimal local supplements. In order to do all this analytically, we assume a general negative exponential distribution for the actions' durations.

Thirdly, summing these local functions for the whole schedule results in a global cost function that can be used by an optimizing solver directly. Minimizing the total objective function over all supplements, respecting all schedule constraints, effectively makes the trade-off between all local supplements.

3 Use of the Objective Function

We show that the constructed objective function can quickly evaluate an existing schedule. Evaluation, even of passenger train timetables for a whole country, takes less than a second. For optimization, we demonstrate that the Belgian schedule with all 288 passenger trains during a morning peak hour, solver times are only about twenty minutes, when knock-on delays between all train pairs per common track sections are ignored, and up to a few hours, when these knock-on delays are considered.

4 Main Contributions

We believe that all factors relevant to determine optimal buffers and supplements for a whole network are present in our objective function and that this is a unique research result. The application of this objective function to optimize the passenger train schedule for a whole country also shows its value in practice.

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